WESLEY COLLEGE
By daring & by doing

eYEAR 12 MATHEMATICS METHODS
SEMESTER ONE 2018
TEST 1 EXPONENTIALS AND LOGARITHMS AND DIFFERENTIAL CALCULUS

Tuesday 27th February

Name: SOLUTIONS

Time: 45 minutes **Part A:** ___ 29 **Part B:** ___ 14 **Total:** ___ 43 %

- Answer all questions neatly in the spaces provided. **Show all working.**
- You are permitted to use the Formula Sheet for both sections, and an A4 page of notes, plus up to 3 permitted calculators in the Calculator Allowed section.

Topic	Confidence
Exponentials and logarithms	
• Logarithm laws and solving equations	Low Moderate High
• Logarithmic graphs and scales	Low Moderate High
• Exponential growth and decay	Low Moderate High
Differential calculus	
• Exponential functions	Low Moderate High
• Natural logarithmic functions	Low Moderate High
• Differentiation rules	Low Moderate High

Self reflection (eg. comparison to target, content gaps, study and work habits etc)

1. **[10 marks]**

Solve the following equations, giving exact answers in simplest form.

a) $2^{x-1} = 7$

$$(x-1)\log_2 2 = \log_2 7$$
$$x = \log_2 7 + 1$$
$$\text{or } = \frac{\ln 7}{\ln 2} + 1$$
$$\text{or } \ldots$$

Take logs Isolate x

[2]

b) $\log_4 x = \frac{3}{2}$

$$4^{\frac{3}{2}} = x$$
$$x = 8$$

Convert to exponential Simplify

[2]

c) $\log_3(3x+1) = 2 + \log_3(x)$

$$\log_3(3x+1) = \log_3 9 + \log_3 x$$
$$3x + 1 = 9x$$
$$1 = 6x$$
$$x = \frac{1}{6}$$

Convert 2 to log statement Log law Equate to find x

[3]

d) $e^{2x} - e^x = 6$

$$(e^x)^2 - e^x - 6 = 0$$
$$(e^x - 3)(e^x + 2) = 0$$
$$e^x = 3 \qquad (e^2 = -2 \text{ no real soln})$$
$$x = \ln 3$$

Factorise Solve for x Acknowledge no further solution

[3]

2. **[5 marks]**

Given that $\log_2 3 = m$ and $\log_2 5 = p$,

a) express $\log_2 30$ in terms of m and p

$\log_2 30 = \log_2(3\times5\times2)$

$= m + p + 1$

Prime factorization of 30
Equivalence in terms of m and p

[2]

b) evaluate 2^{p-2m}

$p - 2m = \log_2 5 - 2\log_2 3$

$= \log_2 \frac{5}{9}$

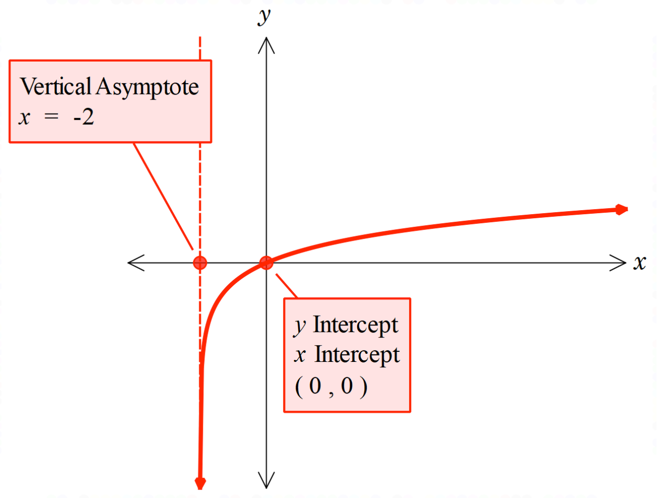
$\therefore 2^{p-2m} = \frac{5}{9}$

p-2m log statements
log law to combine
Evaluate the index form

[3]

3. **[5 marks]**

a) Sketch the graph of $y = \log_2(x+2) - 1$ labelling asymptotes, intercepts and a key point.



Asymptote
(0,0)
Shape

[3]

b) Explain why graphs of equations of the form $y = \log_a(x+a) - 1 \quad a > 0$ always pass through the origin.

$x = 0$

$y = \log_a a - 1$

$= 1 - 1$

$= 0$

i.e. $(0,0)$

Sub x=0
Evaluate for co-ord

[2]

4. **[9 marks]**

a) Differentiate the following equations. Answers should be in the same form as the question but do not need to be simplified or expressed in factored form.

i) $y = \dfrac{x^3}{e^x}$

$$\frac{dy}{dx} = \frac{e^x 3x^2 - x^3 e^x}{e^{2x}}$$

Quotient rule Correct components

[2]

ii) $y = \sqrt{x^2 - 1}\left(5x - x^2\right)$

$$\frac{dy}{dx} = (5x - x^2).\frac{1}{2\sqrt{x^2 - 1}}.2x + \sqrt{x^2 - 1}.(5 - 2x)$$

Product rule Correct components Rational form

[3]

b) Given $y = \dfrac{u^3}{3} - u$ and $u = \ln(2x - 3)$ determine $\dfrac{dy}{dx}$ in terms of x.

$$\frac{dy}{du} = \frac{3u^2}{3} - 1$$

$$\frac{du}{dx} = \frac{2}{2x - 3}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= (u^2 - 1)\frac{2}{2x - 3}$$

$$= \frac{2((\ln(2x - 3))^2 - 1)}{2x - 3}$$

Chain rule Components x 2 Answer in terms of x

[4]

Name: ______________________________

Calculator Allowed Section **15 minutes** /14

5. **[9 marks]**

The size of a population, W, is measured every year and has an instantaneous rate of change given by the equation $\frac{dW}{dt} = \frac{W}{20}$, where t is the number of years after recording commenced.

The initial population is 2500.

a) State whether the population is increasing or decreasing, giving a mathematical reason for your answer.

$$\frac{dW}{dt} = \frac{1}{20}W$$
$$> 0 \quad \text{when } W > 0$$

Hence growth

Increasing Sufficient justification

[2]

b) State an equation for W in terms of t.

Initial value Exponential model

$$W = 2500e^{0.05t}$$

[2]

c) Find the size of W when t = 4. (Round to the nearest integer).

Correct answer

$$t = 4,\ W \approx 3054$$

[1]

d) Find the rate of change of W when t = 4, (correct to 2 decimal places).

$$\frac{dW}{dt} = 125e^{\frac{t}{20}}$$
$$t = 4,\ \frac{dW}{dt} \approx 152.68$$

or

$$\frac{dW}{dt} = \left.\frac{W}{20}\right|_{W=3054}$$
$$\frac{dW}{dt} \approx 152.68$$

Derivative Correct rate

[2]

e) Determine the year in which the instantaneous rate of change first reaches 500 units per annum.

Equation Correct year

$$\text{let } \frac{dW}{dt} = 500 = 125e^{\frac{t}{20}}$$
$$t \approx 27.7$$

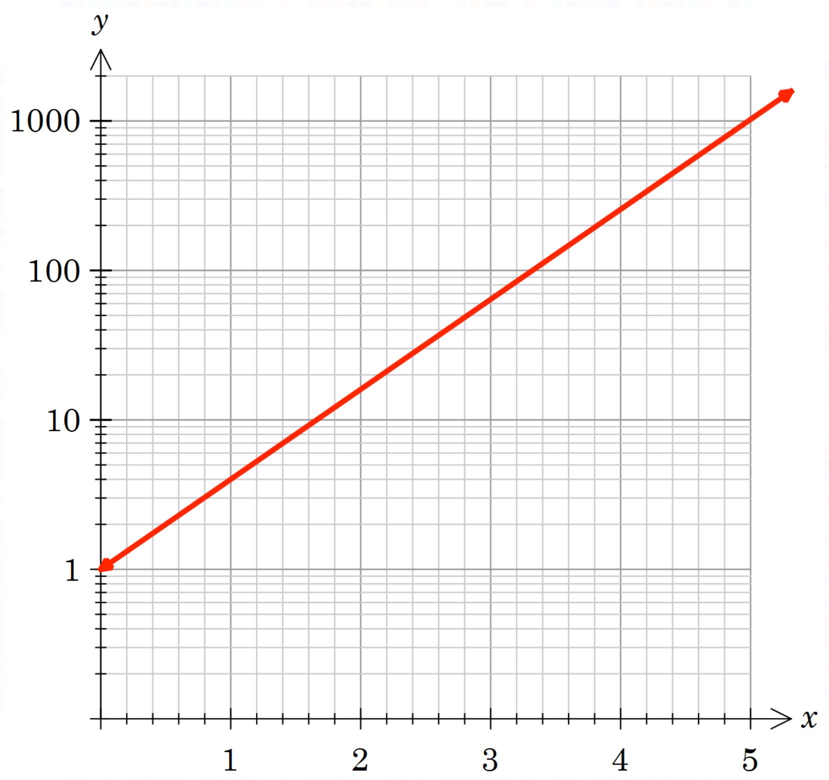
i.e. in the 28th year

[2]

6. **[5 marks]**

a) Plot the function $y = 4^x$ on the axes below.

Note the logarithmic scale on the y-axis.



Linear
Accuracy

[2]

b) The 'db' or Decibel scale for sound level measuring loudness of sound is given by:

$$\text{Sound level} = 10\log\left(I \times 10^{12}\right) \text{ db}$$

Where I is the intensity of the sound in Watts per m^2

Show that doubling the intensity of a sound increases the sound level by only a few db.

let $I_2 = 2I_1$

$$\text{level} = 10\log(2I_1 \times 10^{12})$$

$$= 10\log(I_1 \times 10^{12}) + 10\log(2)$$

$$\approx \text{original level} + 3.01$$

i.e. approximately 3db increase

Substitution
Log law
Correct conclusion

[3]